

Fractal Electromagnetic Showers

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We study the self-similar structure of electromagnetic showers and introduce the notion of the fractal dimension of a shower. Studies underway of showers in various materials and at various energies are presented, and the range over which the fractal scaling behaviour is observed is discussed. Applications to fast shower simulations and identification, particularly in the context of extensive air showers, are also discussed.

1. Introduction

One of the most serious problems in the analysis of cosmic ray data is the complex and time-consuming nature of the codes used for shower simulation. In order to try to capture the detailed physics of the processes involved, it is customary to directly simulate [1,2] the multiplicative branching process whereby an initial particle gives rise to two or more secondary particles, each of which, in turn, initiates what is essentially its own shower, albeit now at lower energy.

Such a process can give rise to large fluctuations, and the final distributions of ground particles and their energies (as well as the longitudinal distribution of the shower as a whole) are difficult to model with simple parametrizations unless one is happy to settle for a description of the *mean* behaviour of the shower and forego knowledge of the fluctuations. Indeed, this is the leading reason that so much Monte Carlo time must be used for shower simulations: there are no simple analytical forms for the relevant distributions which can describe the fluctuations. The issue is a pressing one for experiments collecting large amounts of data which may be difficult to compare against theory in any form other than a large number of simulated events.

Here we report on the observation that electromagnetic showers display self-similar behaviour

which can be described by a multifractal geometry and describe first steps towards formalizing this concept. Our eventual goal is to describe showers in terms of what we argue here is the relevant geometry: not one of smooth functions, but one which allows for irregular geometries which are better described in terms of fractals. We consider here only electromagnetic showers, but plan to study hadronic showers in future work.

2. Self-Similarity in Electromagnetic Showers

The idea that an electromagnetic shower should, in some sense, be a fractal is almost obvious. It is generated recursively from the two processes”

1. pair creation: $\gamma \rightarrow e^+e^-$ in the electric field of a nucleus and;
2. Bremsstrahlung: $e^\pm \rightarrow e^\pm\gamma$ as an electron or positron is deflected by the electric field of a nucleus

This is illustrated in figure 1 which shows the particles making up a shower produced by a 100 GeV electron entering a block of aluminum 150 cm long (radiation length 8.9 cm) as simulated using the GEANT4 program [3].

Each final state particle from an interaction effectively initiates its own electromagnetic shower, and each process has a similar cross section to occur in matter. As long as the energies involved are large compared with the energy required to create an electron-positron pair (and thus also large

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compared to atomic processes such as ionization), each step is much the same as the one before it, but at a reduced energy.

Figure 2 shows a slice through the block right at the far end with the point of intersection of each particle with the slice shown as a black dot whose radius is independent of energy. Here one clearly sees the shower core, with a diminishing density of particles with distance from the centre.

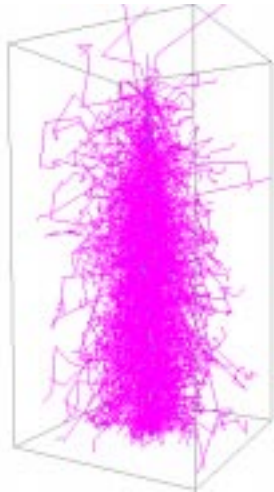


Figure 1. Three-dimensional view of the tracks making up an electromagnetic shower due to a 100 GeV electron entering an aluminum block 150 cm in length.

3. Fractals and Multifractals

There are many ways to characterize self-similar objects, but the most common and well-known way is in terms of fractal dimensions. There are many different concepts of fractal dimension which are useful, and perhaps the most obvious is that of mass dimension, D_M . The idea here is to see how the total energy $E_{TOT}(R)$ (considered now as a sort of weight) within a disk of radius R varies as R is changed. If the distribution were one-dimensional (a line of uniform energy deposited in the plane), one would find

$$E_{TOT}(R) \propto R^1 \quad (1)$$

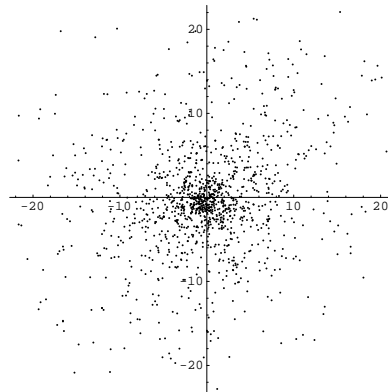


Figure 2. Points of intersection of particles with a slice 150 cm from the entry point of a 100 GeV electron into an aluminum block. Axes are perpendicular to the shower axis and are marked in centimetres.

and one would take the exponent in the foregoing equation to be the dimension of the distribution.

If the energy were uniformly distributed over the whole plane, one would find

$$E_{TOT}(R) \propto R^2 \quad (2)$$

and conclude again that the exponent in the scaling law for the energy should be interpreted as the dimension of the distribution.

In the event that a scaling law of the form $E_{TOT}(R) \propto R^{D_M}$ holds for a non-integer D_M , we call D_M the “fractal mass dimension”. A plot of $\log(E)$ as a function of $\log(R)$ will then have a slope in the limit of small R which is D_M .

Two points are important to keep in mind here: first that there are no true fractals in nature as there are always some smallest and largest value for variables in the problem beyond which scaling behaviour does not hold, and second that one must be careful to watch for systematic effects which can bias estimates of the dimension. Systematic effects which we have had to be wary of include the fact that early in the shower development the central core can contain particles which

carry a large fraction of the initial energy and give the radial energy distribution a spike at small R which does not correspond to scaling behaviour.

In the case of the electromagnetic shower with the slice taken at the end of the shower at 150 cm, we look at the summed energy (scaled so that the total energy is 1) as a function of the fraction of the radius out (scaled so that the maximum radius is 1). This quantity we denote as $I(R|1)$ for reasons which will become clear later in the text. Plotting logarithms against logarithms (base 10), we find the distribution shown in figure 3. The

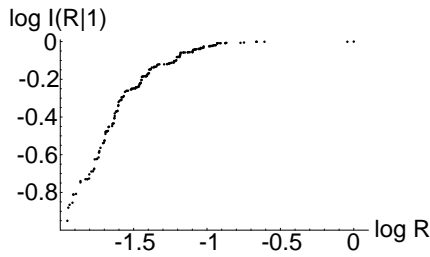


Figure 3. Summed energy as a function of radius from the shower centre. Logarithms are base 10.

first thing to notice is that the curve is reasonably approximated by a straight line at small radii. The second thing to notice is that the whole curve is *not* a straight line. At large radii we start to reach the physical boundaries of the shower and cannot expect scaling to hold.

In fact, even at very small radii, there is some anomalous structure which can be traced to the effects of very energetic particles very close to the core, which give an additional spike of energy to the distribution which cannot be expected to be a part of any overall scaling behaviour. This effect is more pronounced earlier in the shower.

The scaling properties of the shower are thus different in different parts of the plane, and in order to quantify this further, we study the scaling behaviour of cumulative moments of the energy

distribution defined for $q > 0$ by

$$I(R|q) = \frac{\sum_{r < R} E_i^q}{\sum_{\text{all } i} E_i^q} \quad (3)$$

where E_i are the energies contained in a disk going out to radius R and the sum is taken over all particles within a distance $r < R$. What units are used is not important as we are only interested in the average scaling behaviour of the curves at small $R \rightarrow 0$. (As discussed earlier in the text, the region of very small R should be avoided for physical reasons, and we will avoid the subtleties of precise numerical analyses in this short communication.) For graphical purposes here, R is normalized so that the particle with the largest radial distance out is at $R = 1$ and the moments are defined so that their value at maximum radius is unity. We can then introduce an infinite family[4] of fractal dimensions D_q defined for $q > 0$ by

$$D_q = \lim_{R \rightarrow 0} \left\langle \frac{1}{q} \frac{\partial \log I(R|q)}{\partial \log R} \right\rangle \quad (4)$$

with the understanding that the limit must still lie in the scaling region in physical examples.

Figure 4 shows the scaling behaviour of moments of the electromagnetic shower corresponding to how the sums of the squares and cubes of the energy grow with distance. For a homogeneous and uniform fractal structure we expect the D_q to be equal. If not, then we describe the distribution as multifractal in that it requires more than one fractal dimension in order to characterize it. The associated D_q for small q estimated from finite differences in the scaling region are all approximately equal within the errors in the data here and approximately unity, suggesting a good degree of homogeneity. It is important to keep in mind that the results in this paper are presented for a full, realistic GEANT simulation, and include ionization, delta-ray, and other soft processes, so some care is needed in interpreting the results as if they corresponded to a pure electromagnetic shower generated only by pair creation and Bremsstrahlung (which is, of course, not realizable in nature).

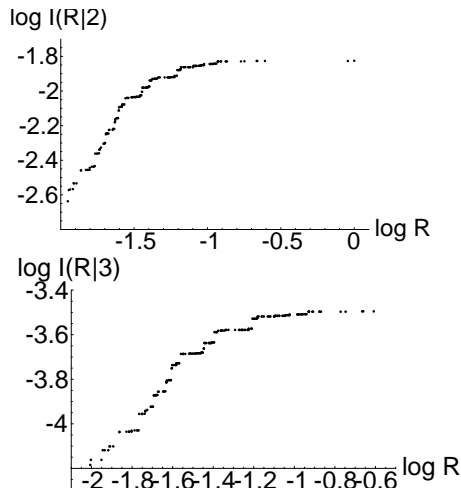


Figure 4. Total energy-squared (above) and total energy-cubed as functions of radius from the shower centre (see text for normalizations). Logarithms are base 10.

The definition of fractal dimensions can also be continued to $q \leq 0$, but this has some subtleties involved with the fact that as $q \rightarrow \infty$ the highest energy particles contribute most, while as $q \rightarrow -\infty$ the lower energy ones dominate. In particular, some care must be used with the D_q for $q < 0$ as they give high weights to softer particles which are not part of the hard shower process. These matters, as well as more precise results on dimensions including energy and material dependence will be presented elsewhere[5].

4. Further Work

Clearly space limitations make it impossible to cover the material as completely as one would like, but several points concerning work not discussed here are worth making. First of all, we expect fractal behaviour in all three dimensions, and in this discussion we have neglected the longitudinal scaling behaviour, where the full shower is made of many scaled and translated showers superimposed along the shower axis. In addition,

there are clearly angular correlations and fluctuations, and studies can be made at a given fixed radius of the scaling behaviour of the shower as a function of the angular coordinate which we have integrated out in this discussion. The relation of these ideas to the concept of intermittency, especially as studied in hadronic jets has not escaped our notice and is currently under investigation.

One of the main goals of this work is to better understand the geometry of electromagnetic (and other) showers in order to try to parametrize them by the appropriate non-smooth basis functions, such as wavelets. Such a parametrization should allow the fast generation of showers without the attendant loss of information concerning large fluctuations[5] which has so far been handled only by the use of enormous computational resources.

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